Closed Symbolic Execution for Verifying Program Termination

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12th IEEE Int'l Working Conf. on Source Code Analysis and Manipulation
SCAM 2012

Sep 23-24, 2012
Riva del Garda, Italy

Motivation

Using symbolic execution for proving program termination
(and other liveness properties)

- there are already a few ad-hoc approaches: AProVE, Costa, Julia (Haskell, Prolog, Java bytecode)
- we aim at presenting a higher-level, language independent scheme using well-known notions and techniques from partial evaluation

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Symbolic execution: example

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l_0: & x := \textbf{input}(); \\
l_1: & \textbf{while } x > 0 \textbf{ do} \\
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\langle l_1, \{x \mapsto 1\}\rangle \\
\downarrow \\
\langle l_2, \{x \mapsto 1\}\rangle \\
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\langle l_1, \{x \mapsto 0\}\rangle \\
\downarrow \\
\langle l_3, \{x \mapsto 0\}\rangle
\end{array}
\quad \Rightarrow \quad
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\langle l_3, \{x \mapsto X\}, \neg(X > 0)\rangle & \langle l_2, \{x \mapsto X\}, X > 0\rangle \arrow[d] \\
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This work

- Use symbolic execution to overapproximate standard executions (as in partial evaluation)
- Use the symbolic execution graph for verifying program termination (or other liveness properties)

In particular,

- we produce a term rewriting system that represents the transitions of the symbolic execution graph (as in partial evaluation)
- and apply existing termination provers for term rewriting systems

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Terminating! (using AProVE)

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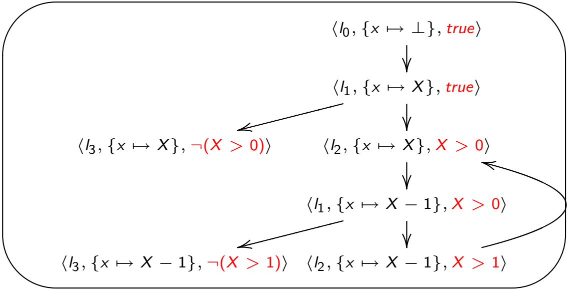

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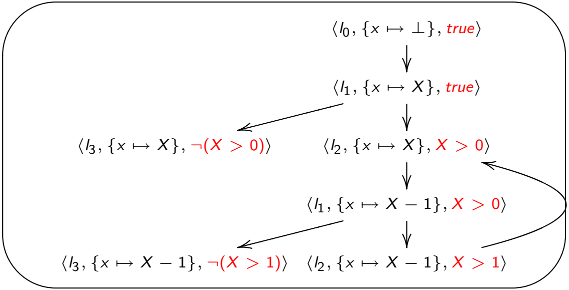

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Proof-of-concept implementation

SETT (Symbolic Execution-based Termination Tool)

- simple imperative programs with integers, basic arithmetic, assignments, conditionals and jumps

web interface: http://kaz.dsic.upv.es/sett/

Conclusions

- Powerful scheme for proving program termination
- Same scheme can be used
  - for other (dynamic) programming languages (Erlang, JavaScript)
  - for other (liveness) properties